CHAPTER 17
REFLEXIVE VARIABLE

17.1. Reflexivity is a manifest case of indexicality and represents an insidious theme that I intend to analyze in detail, owing to its primary importance for the history of logic.

17.2. Let *L* be a symbolic language and *ML* its metalanguage. In *L* *"a"* and *"b"* are individual constants (for "Ava" and "Bob"), *"x"* and *"y"* are individual variables, *"A"* and *"B"* are verbs (for "to admire" and "to blandish"). In *ML* "**Q**" "**R**", "**T**" and "**Φ**" are (syntactical) variables respectively for monadic predicates, dyadic predicates, terms and monadic functors. For instance, then, the *L*-translation of

(17.i) she admires him

is

(17.ii) *A(x,y)*

(she who? he who?). Analogously

A(x,b)

is the *L*-translation of

(17.iii) she admires Bob

(she who?) and

(17.iv) *A(a,y)*

is the *L*-translation of

(17.v) Ava admires him

(him who?). Here too (§15.14.1) the discourse is valid until the sentences are considered separately. In fact as soon as they are inserted in a wider context where the pronouns have an opportune antecedent, the situation changes. For instance the correct *L*-translation of

(17.vi) if Ava blandishes Bob, then she admires him

is

$B(a,b) \rightarrow A(a,b)$

because what (17.vi) tells us is exactly that she is Ava and he (him?) is Bob. Therefore

A(a,b)

is now the correct *L*-translation of the same (17.i) whose previous correct *L*-translation was (17.ii).

I insist in emphasizing that the aim of the syntactical approach is to comply with the current ones. The very core of the discourse continues being semantic. If I utter (17.i) when my interlocutor and I are looking at Ava and Bob chatting pleasantly on a desert shore, the conversion of the two variables is performed without any grammatical antecedent because the anaphoric function finds the integrative informational source in the two persons we are looking at.

17.3. The theoretically crucial passage is that while natural languages do possess the possibility to formulate directly reflexive relations, current symbolic languages do not; and though at a first and superficial sight such an impossibility may seem non-reductive, it actually mutilates the expressive power of the languages in question because reflexive predicates singularly considered become unattainable.

Even the well-developed natural languages do not represent a sophisticated linguistic model; wants, ambiguities and redundancies are too many. On the other hand the well-developed natural languages are a highly tested means of communication, and anyhow they represent the best model at our disposal. Of course among well-developed natural languages many expressive differences do exist; yet the common presence of some basic characteristics in all of them (from Port Royal to Leibniz and to the same Chomsky) suggests the presence of some universal requirements dictated by a universal *modus cogitandi.* As far as I know reflexive variables occur in every well-developed natural language; and as such we either arrogantly ignore the result of a linguistic experience ripened in thousands of centuries, or we have to acknowledge the opportunity of their presence in artificial symbolic languages too.

17.4. While we can easily and unproblematically symbolize

(17.vii) Ava admires herself

by

(17.viii) *A(a,a)*

as soon as we try to symbolize separately the reflexive predicate

(17.ix) to admire oneself

we realize immediately that the lack of a reflexive variable actually mutilates the expressive power of the same symbolic language. In fact

A(...,x)

symbolizes

to admire the person to specify

and

A(...,a)

symbolizes

to admire Ava

(just because in (17.viii) "*a*" occurs both as the subject and as the complement, the same (17.viii) can symbolize (17.vii); but this procedure by iteration, obviously, fails where, as in (17.ix), we deal with a reflexive predicate alone).

In order to overcome this heavy limit I introduce in *L* a reflexive individual variable *"s"* (or even, for the sake of generality, a set of reflexive variables *"s_1", "s_2"* et cetera) whose conversion rules will be discussed below. For the moment I legitimate its right to exist by remarking that

A(...,s)

symbolizes exactly (17.ix). Analogously, for instance,

A(a, s_1) & A(b, s_2)

says (roughly) that both Ava and Bob are narcissists (Ava admires herself and Bob admires himself).

17.4.1. With reference to §16.6,

$d_{\beta(s)} > 500$

is the correct symbolization of "peripheral"; but without "*s*", which assures the right conversion of the indexical predicate into the contingent absolute ones ("far from Madrid" if the subject is "Cadiz", "far from Rome" if the subject is "Trieste" et cetera), how could we carry out such a symbolization?

To realize such an impossibility is to realize that the introduction of reflexive variables is a due (and very useful) enrichment of symbolic languages.

17.5. Reflexive and non reflexive (or generic) variables constitute two sorts of individual variables, and the theoretically discriminating factor between them is that they obey different conversion rules.

Here is an easy example. The only difference between

(17.x) When Ava is ill, Eve takes care of her

and

(17.xi) When Ava is ill, Eve takes care of herself

concerns the two pronouns: generic in (17.x), reflexive in (17.xi). The respective messages are manifestly different: while (17.x) suggests a charitable Eve, (17.xi) suggests a neurotically egoistic Eve. And this difference follows from the different antecedents of the two pronouns (variables): while "her" stands for "Ava", "herself" stands for "Eve". Then we must distinguish between the anaphoric function ruling generic indexical variables, and the anaphoric function ruling reflexive variables. Here too (§16.5.2) we can avoid the detailed theorization of the conversion rules for reflexive variables, since the sentences of our very interest are not affected by any problem of conversion. In fact it is sufficient to agree that where the reflexive variable occurs in the predicate of an atomic sentence, its antecedent is the respective subject.

17.5.1. A subtle question runs as follows. In (17.x) no reflexive pronoun occurs, and the syntactically possible antecedents of "her" are "Ava" and "Eve" (which, furthermore, is the nearest one); why on earth, then, when we read (17.x) do we all understand unequivocally that the right antecedent is "Ava"?

The answer is simple. Since we all know that English possesses reflexive pronouns, we also know that in order to assume Eve as referential antecedent we should use "herself"; then, since in (17.x) a non-reflexive pronoun is used, its referential antecedent is not Eve. In other words, Eve is a no longer available antecedent for the non-reflexive anaphoric function.

17.5.2. Deep searches on the notion of reflexivity are not necessary to understand that, singularly considered, a generic variable (*the individual to specify*) adduces less information than a reflexive one (*the individual we are speaking of*). Here is the simple reason why (I follow Quine's symbolism in *Methods of Logic*, §25)

$Gyy \supset \exists x(Gxy)$

is a valid scheme, while

$Gxy \supset \exists x(Gxx)$

is not. In other words: since *that same individual* implies *an individual* but not vice versa, the truth of

(17.xii) *Gyy*

is sufficient to assure the truth of

(17.xiii) $\exists x(Gxy)$

but the truth of (17.xiii) is not sufficient to infer the truth of (17.xii). In this sense here too there is no need to adopt integrative conditions (which, moreover, are not supported by theoretical arguments but only by counter-examples, as for instance in the quoted Quine's work).

17.6. While the examples above concern the direct reflexivity,

(17.xiv) Ava admires her own mother

concerns an indirect reflexivity. Nevertheless the difference is nearly negligible. In fact, since (17.xiv) can be symbolized in something like

(17.xv) $A(a, \mu(s))$

both the direct and the indirect reflexivity enter into the general *ML*-scheme

(17.xvi) **R(T, Φ(**"*s*"**))**

as soon as the direct reflexivity is conceived as the particular case where **Φ** is the null functor of identity. That is: the theoretically essential achievement is not the absence of a functor in

(17.xvii) $A(a, s))$

but the necessary presence of the reflexive variable both in (17.xv) and (17.xvii).

17.6.1. Reciprocity too is a case of indirect reflexivity. For instance

(17.xviii) Ava and Bob greeted each other

means that Ava greeted her greeter and Bob greeted his greeter, then (17.xviii) too enters into the scheme (17.xvi).

The same conclusion is valid also with reference to an *n*-adic circularity.

17.7. An easy application of the reflexive variable allows us an immediate solution of Kripke's paradox. Its well known derivation runs as follows. Since

$\forall x \forall y ((y=x \ \& \ Px) \supset Py)$

and since

$\forall x(x=_{nec}x)$

(where "$=_{nec}$"symbolizes the relation of necessary equality), then, particularizing *P* on the predicate

(17.xix) *"*$=_{nec}x$*"*,

we get

$\forall x \forall y ((y=x \ \&(x=_{nec} x)) \supset (y=_{nec}x))$

and finally

$\forall x \forall y ((y=x) \supset (y=_{nec} x))$

a conclusion in manifest contrast with our common sense.

The immediate solution points out that the actual predicate is not (17.xix) but

(17.xx) $=_{nec}s$

(actually every individual is necessarily equal to himself or herself or itself, not necessarily equal to *x*): and (17.xx) is an indexical predicate that by *s*-conversion becomes *"*$=_{nec}y$*"* when it is ascribed to *y*, just as it becomes *"*$=_{nec}x$*"* when it is ascribed to *x*.

17.7.1. The ordinary solution of Kripke's paradox points out that (17.xix) is not a correct particularization of *P* because it binds *"x"*. Yet such a solution is affected by a strong theoretical fault since (§15.15) just under the current orthodoxy, the incorrectness of such particularizations is not a theorem, but only an assumption grounded on counter-examples.

17.8. I think that the very root of the misleading approach to logical paradoxes is the current classification of open sentences. In compliance with a famous Skolem's suggestion (1922), the classifying criterion is the number of different free variables occurring in the open sentence under scrutiny (monadic, dyadic et cetera), without minding the number of occurrences for every free variable. Not to overcharge the analysis, I only consider sentences in model form, once agreed that a monadic sentence is in model form iff

- its subject consists of the simple free variable
- its predicate is either free-variable-free or free-variable-laden

(of course in a monadic sentence whose subject is free-variable-laden, either the predicate is free-variable-free or its variable must be the same of the subject, therefore in this case the sentence is reflexive).

Considering only sentences in model form is not so reductive a simplification as it may seem. For instance,

(17.xxi) Ava's mother was run over by the eldest sister of his wife

is a monadic sentence (the wife of whom?) whose form is far from the model one; yet it is easy to transform (17.xxi) into

(17.xxii) he is the husband of a lady whose eldest sister ran over Ava's mother

and to ascertain that (17.xxii) is in model form. Anyhow, even if the simplification were highly reductive, it does not forbid me to tackle the logical paradoxes and to give them a general solution.

17,8.1. I provisionally epitomize these conditions in the symbolic scheme

(17.xxiii) **P(V)**

where **V** (without inverted commas, I am using the metalinguistic symbol to speak of object expressions) is the *L*-variable (as *"x", "y"* et cetera) constituting the subject of the model sentence, and **P** (idem) is a *L*-predicate either free-variable-free or free-variable-laden.

17.9. The crucial achievement is that even under these simplifications, Skolem's criterion is highly unsatisfactory: in fact it does not introduce any distinction between monadic sentences like (17.xxii), whose predicate

(17.xxiv) being the husband of a lady whose eldest sister ran over Ava's mother

is free-variable-free (absolute), and model sentences like

he is the husband of a lady whose eldest sister ran over his mother

whose predicate

(17.xxv) being the husband of a lady whose eldest sister ran over his mother

is free-variable-laden and more precisely reflexive.

17.10. Exactly because both (17.xxiv) and (17.xxv) enter into the provisional scheme (17.xxiii), such a scheme does not account for the basic distinction between free-variable-free and free-variable-laden predicates. In order to overcome such a mutilating inadequateness, once assumed "**C**" as a metalinguistic symbol over *L*-constants, I oppose the scheme

(17.xxv) $\mathbf{P}_{\mathbf{\Phi(C)}}(\mathbf{V})$

or indifferently

(17.xxvi) $\mathbf{R}(\mathbf{V},\mathbf{\Phi}(\mathbf{C}))$

to the scheme

(17.xxvii) $\mathbf{P}_{\mathbf{\Phi}(\text{"}s\text{"})}(\mathbf{V})$

or indifferently

(17.xxviii) $\mathbf{R}(\mathbf{V},\mathbf{\Phi}(\text{"}s\text{"}))$

so overcoming the ambiguity of (17.xxiii), where **P** **may be a free-variable-free as well as a free-variable-laden predicate** without compromising the monadic character of the respective sentence.

17.11. Until now I have spoken of sentences, but henceforth I prefer to speak of dilemmas because I think that it should be better and clearer. So, for instance, I will say that while (17.xxv) and (17.xxvi) are schemes of non-reflexive (absolute) dilemmas, (17.xxvii) and (17.xxxiii) are schemes of reflexive dilemmas. And incidentally I remark that paradoxical and anti-paradoxical (and para-paradoxical) dilemmas belong to the latter case.