

CHAPTER 4

DEFINITIONS

4.1. The usual acceptation of “to define” is very wide, since we can define the rules of a play, the boundaries of a State, the powers of an office et cetera. In this sense whatever intervention adducing sufficient information to identify a certain referent is a definition. Even a jaguar marking its territory is defining it. Fortunately we can avoid being involved in urological practices, since our interest is focused on definitions concerning the information adduced by a linguistic expression. This notwithstanding, under the informational approach (Suppes 1957 §8.2)

(4.i) *A definition is a statement which establishes the meaning of an expression*
 is too restrictive a definition of -definition-. In fact the wider viewpoint of the informational approach can easily concern definitions which are not statements. For instance the young mother uttering “hat”, “hat”, “hat” while showing different hats to her son is evidently performing a definition.

The crucial requisite is supplying the information through which a meaning is assigned to a definiendum, not the means (linguistic or ostensive) adopted in order to supply such an information. However linguistic definitions will be henceforth privileged.

4.2. Assuming (roughly)

(4.ii) $x\sigma y$
 as the general scheme of a definition, is assuming that the first variable ranges over expressions (of a given language L σ refers to) and the last variable ranges over meanings (a more detailed analysis in §4.6)). The informational levels involved by (4.ii) are evidently two: one for the object piece of information y (that is the piece of information which by definition is adduced by the definiendum x), and one for the piece of information adduced by the whole (4.ii), that is the piece of information telling us that (in L) the object piece of information y is adduced by the expression x . Therefore a definition is an intrinsically hyperlinguistic intervention. A due conclusion whose insidiousness is evidenced by the acritical resort to non-synonymous verbs (as -to be-, -to mean-, -to stand for-, -to be equivalent to- et cetera), or even to different symbols (as “ \rightarrow ”, “ $=$ ”, “ \equiv ”)) in order to express the relation between a definiendum and its definiens.

4.2.1. In order to avoid misinterpretations, let me recall some previous assumptions.

The σ -relation between a sign (strictly: between the image of a sign) and the piece of information it adduces (§1.6) is intrinsically mental: no σ -relation without a knower.

The σ -relation is conventional; it depends on a code, usually the code of a public language,

The linguistic ability is the mental faculty of establishing a net of (conventional) σ -relations.

The mental intervention necessary to abstract a type from tokens is absolutely distinct from the mental intervention necessary to associate a certain meaning to a certain sign. The latter, so to say, is the second stage of a procedure whose first stage is just the abstraction of the type (otherwise every occurrence of a token should demand a specific definition).

4.2.2. The mental character of the σ -relation is a claim subject to the following (and superficial) objection.

The physical contiguity between the golden

(4.iii) BEATRICE
 finely painted on its stern makes (4.iii) the name of this same yacht, quite independently of any mental intervention.

Reply. Their physical contiguity is nothing but an ostensive informational source telling us that actually the (image of the) name and the (image of the) yacht are connected by a σ -relation. Yet identifying the physical relation of contiguity between the painted name and the yacht with the semantic one, is a mistake: also the golden arabesques finely painted to enclose (4.iii) are physically contiguous to the yacht, nevertheless they do not name anything, simply because they are not interpreted as meaningful marks (*i.e.*: simply because we assume that their image is not the origin of any σ -relation).

4.3. In order to classify formal definitions let me propose a fanciful example.

By supposition the worldwide envied lady who owns the Parthenon, owns the Coliseum too. This piece of information can be indifferently adduced by

(4.iv) The Coliseum is owned by the owner of the Parthenon

or by

(4.v) The owner of the Parthenon is the owner of the Coliseum

or by

(4.vi) The Coliseum and the Parthenon are co-owned

(of course two or more things are co-owned iff they belong to the same owner(s)).

Nevertheless while (4.iv) is centred on a relation (“to be owned by”) between a monument and an owner, (4.v) is centred on a relation (“to be”) between owners, and (4.vi) is centred on a relation (“to be co-owned”) between monuments.

The three formulations above correspond respectively to the three strict linguistic formulations of a definition (where monuments become signs and owners become meanings). In fact a linguistic definition can be strictly formulated

- by signification (§4.4)
- by semantic identity (§4.5)
- by synonymy (§4.6)

(I speak of strict formulations because (§4.8) there are linguistic formulations whose hyperlinguistic dimension is not explicit).

4.4. The scheme

(4.vii) "....." σ *.-.-.*

that is, for instance,

(4.viii) “regular (polygon)” means *equilateral and equiangular (polygon)*

illustrates the simplest version (§4.4.3) of definitions by signification. Both the presence of “ σ ” and of asterisks make manifest the hyperlinguistic dimension of (4.vii).

The formulation by signification (which anyhow can be immediately and plainly translated into a formulation by semantic identity or by synonymy) plays a privileged role since it respects in the most scrupulous manner the task of assigning a meaning to a definiendum (4.i).

A fastidiousness. In (4.viii) “polygon” occurs between parentheses because when we do not speak of polygons but, say, of soldiers, “regular” does not at all mean *equilateral and equiangular*. Another fastidiousness: “polygon” has been and will be used as an abbreviation of “plane polygon with rectilinear sides”.

4.4.1. Since σ is a projective relation (that is a relation connecting a syntactic entity with a semantic one), a semantic diacritical symbol as asterisks is necessary to write down a correct formulation by signification. Two illustrious examples of the impasse otherwise impending are given by Pap, where he writes

(4.ix) “ $f'y$ ” means “the number which is the immediate successor of y ”

(1964, in Olszewsky 1969, p.287), and by Tarski, where he writes

(4.x) *It might appear ... that “true sentence” ... means nothing other than “provable theorem”*

(1936, §3); they did not realize that a scheme like

(4.xi) “ x ” means “ y ”

is either ambiguous or incoherent. In fact I remind the reader that the only way to save (4.xi) from incoherence is to accept quotation marks as an ambiguous symbol which in its first occurrence must be interpreted as a name of the enclosed expression and in its second occurrence must be interpreted as a name of the piece of information adduced by the enclosed expression. Yet, also leaving out of consideration that neither Pap nor Tarski point out the problem, the ambiguous agreement would represent however a very unprofitable way out: logic and ambiguity are (at least: ought to be) mortal enemies. In this sense, frankly, I think that (4.ix) and (4.x) are further evidence of the already denounced insufficient dialinguistic perspicuity.

Let me indulge in a frivolous analogy. Just as it would be a very hard task to put a flattened top hat on before the magic touch of the *viveur* restores its third dimension, it is a very hard task to solve enormous problems of logic (one for all: the problem of truth) with a diacritical symbology unable to account for their informational dimension.

4.4.2. The explicit formulation of (4.vii) in ordinary language is

(4.xii) “...” means the meaning of “.-.-.”

and (4.xii) may ring like a false note; yet this remark concerns only the elegance of the sentence. In fact, if what a human being writes is a writing and what a machinery produces is a product, what an expression means is a meaning; then (4.xii) is perfectly proper. The conclusion can be confirmed by instancing (4.xii) on an example where both couples of quotation marks enclose the same expression (“equilateral” means the meaning of “equilateral”, say): the definition becomes a mere tautology, but a tautology entails automatically its properness. Moreover if we substitute “means” by “adduces”, (4.xii) does no longer ring like a false note since the inelegant repetition is avoided.

4.4.3. Indeed (4.vii) is a simplified scheme. The general scheme is

(4.xiii) “...” σ_{L1} *.-.-.* $L2$

where $L1$ is the language the established signification belongs to, and $L2$ is the language whose semantics is used to identify the meaning adduced by the definiens; then (4.vii) is a simplified scheme because, under the implicit assumption $L1=L2$, the indexes are omitted.

In order to write down a definition by signification, the basic problem is finding out the (three) expressions through which the identification of the (three) elements occurring in (4.ii) can be achieved. As for the definiendum, under the standard criterion (§3.6) it is sufficient to enclose the same definiendum within quotation marks. As for the relation, “ σ ” is the obvious solution. But as for the definiens, its identification implies the appeal to another signification (not necessarily belonging to the language of the definieendum). So for instance the understanding of (4.xiv) “casque” means in English “parte della armatura a protezione della testa”, presupposes the understanding of the pieces of information adduced in Italian by “parte”, “della” et cetera.

Yet a doubt may arise as soon as

(4.xv) (4.xiv) defines -casque-

is recognized as a proper (and true) proposition: in (4.xv) what specific dialinguistic symbol hyphens do stand for? The answer is clear: since (4.i) defining is establishing the meaning of an expression, hyphens stand for quotation marks. In fact if we should read (4.xv) as

(4.xiv) defines *casque*

the same definition of asterisks (§2.4) would lead us to

(4.xiv) establishes the meaning of the meaning of “casque”

that is to an improper sentence.

Therefore also in the informational approach what we define are words, not meanings.

4.4.4. Ostensive definitions can be easily interpreted as definitions by signification. For instance, in the case mentioned in § 4.1, the definiendum is uttered (“hat”, “hat”, “hat”), the meaning is inferred by an abstraction on the common connotation characterizing the various exhibited objects (roughly: all of them are covering for the head), and the σ -relation linking the uttered word with this inferred meaning results from the context. Although affected by informal factors (the hyperlinguistic intervention follows from an intuitive association) and by intrinsic limits (how could the young mother define *otherwise* through an ostension?), ostensive definitions represent the basis of every natural semantics.

4.5. The scheme

(4.xvi) $*...* = *_{-.-}*$

that is, for instance,

(4.xvii) $*\text{regular (polygon)}* = *\text{equilateral and equiangular (polygon)}*$

illustrates the formulations by semantic identity in its simplest version. In fact (4.xvi) says that the definiendum and the definiens adduce the same piece of information (therefore “=” occurs in (4.xvi) as an unobjectionable symbol of identity). Of course

(4.xviii) the meaning of “regular (polygon)” is the meaning of “equilateral and equiangular (polygon)” is the explicit formulation of (4.xvii). The hyperlinguistic dimension of (4.xvi) and (4.xvii) results from the occurrence of asterisks, just as the hyperlinguistic dimension of (4.xviii) results from the occurrence of “meaning”.

4.5.1. Exactly as (4.xiii) is the general version of (4.vii)

(4.xix) $*...*_{L1} = *_{-.-}*_L2$

is the general version of (4.xvi).

4.5.2. What I stated in §4.4.1 with reference to formulations by signification can be re-proposed with reference to formulations by semantic identity, since if our endowment of diacritical symbols were limited to quotation marks, a scheme focused on the relation of identity would be either incoherent or ambiguous (§4.6.2).

4.6. The scheme

(4.xx) $“...” \equiv_{\sigma} “-.-”$

that is, for instance

“regular (polygon)” is semantically equivalent to “equilateral and equiangular (polygon)”

illustrates the definitions by synonymy in its simplest version. In (4.xx) “ \equiv ” is a symbol of equivalence and the index “ σ ” tells us that the equivalence concerns the semantic dimension (a synonymy is nothing but a semantic equivalence). So, since inverted arrows have been introduced as a symbol of synonymy (§2.10.1),

(4.xxi) $“...” \Leftarrow “-.-”$

is the alternative formulation of (4.xx) which will be privileged. Here I do not dwell on the relations of equivalence, which will be analyzed in the course.

4.6.1. I recall §4.4.3 and §4.5.1 to underline that (4.xxi) is a simplified scheme; in fact

(4.xxii) $“...”_{L1} \Leftarrow_{L2} “-.-”$

is the general scheme applicable where the two expressions belong to different linguistic codes. Yet, for sake of concision, wherever possible I will reason on the simplified schemata (which obviously hold in formal systems, where we act within an only language).

Once we agree that in its technical acceptation an abbreviation does not imply any shortening component, we can also call “by abbreviation” this kind of definitions; this notwithstanding, though the definiendum of a definition by abbreviation may also be longer than its definiens, it is usually shorter (mainly because if we already have a shorter expression for a meaning, it would be an anti-economic intervention to introduce a longer one).

4.6.2. In the current treatments, strict formulations as (4.xiii) and (4.xxii) are often replaced by (4.xxiii) “...” = “.-.”

that is by a hybrid scheme which, once more, compels celebrated authors to untenable devices. In fact a point is sure: except the trivial case of a tautology, the two expressions enclosed within quotation marks in a particularization of (4.xxiii) are different, therefore the identity symbol “=” cannot be read as an identity symbol. Actually, for instance, Carnap reads it as “to be interchangeable with”, and Church (in Runes) as “to stand for”. But consequently and evidently, since elsewhere “=” continues adducing identity, these readings make it an ambiguous symbol and, moreover, an ambiguous symbol whose secondary meaning ought to be formally defined. On the contrary

- in (4.xvi) “=” occurs properly, because in (4.xvi) it actually adduces identity (of meaning)
- in (4.xix) “↔” can properly be read as “to be interchangeable with” or as “to stand for” or as “to be an abbreviation of” because, once asterisks have been introduced in the formal system, such readings are exactly entailed by the definition of inverted arrows (§2.10.1).

4.6.3. An important note. The fact that in (4.xxi) neither asterisks nor “σ” occur does not at all mean that, therefore, a definition by synonymy, far from being hyperlinguistic, is simply metalinguistic; the hyperlinguistic dimension of (4.xxi) hides in inverted arrows (§2.10). Accepting the reciprocal substitutability of two signs without imposing the identity of meaning (in the same interpretation) would be a ruinous step for the consistence of whatsoever theory. In order to support this conclusion it is sufficient to evoke the admissibility criterion (Salmon 1966 p. 64 quoted in Hajek 2003, §2): *the meanings assigned to the primitive terms ... transform the formal axioms, and consequently all the theorems, into true statements*. In other words: the risk connected with Carnap’s and Church’s position is not to realize that *to be interchangeable* or *to stand for* mask under their syntactic pertinence a semantic regard.

Let me insist. In a formal system, definitions are simply the means to introduce non-primitive symbols as abbreviations of primitive or previously defined symbols; therefore the strict formulation of such definitions is (4.xxi). But as soon as we interpret “symbol” (or “sign”) in compliance with its strict acceptation, the semantic regard becomes evident, since a symbol is something which belongs to a language, and a language is intrinsically an instrument to communicate information. Thus, though a formal system leaves any specific interpretation out of consideration, two expressions, in order to be reciprocally substitutable, must adduce the same meaning; otherwise the admissibility criterion might be respected by one of them and violated by the other one. Shortly: in whatever formal theory whatever abbreviation entails an interpretative equivalence.

The understanding of this claim might be helped by reconsidering (4.vi); it speaks of monuments, but the involved (cadastral) relation is ‘semantic’, as it implies the figure of an owner. In this sense, as soon as we realize that in *sign* the semantic dimension is essential, we realize that “to be synonymous” expresses a relation of semantic regard in spite of its syntactic pertinence.

4.7. Let me spend two lines apropos of a coarse puzzle concerning definitions. On the one hand a definition adduces a new piece of information (without it, obviously, we could not manage the definiendum); on the other hand since (I am re-quoting Suppes) *a new definition does not permit the proof of relationship among the old symbols which were previously unprovable*, the same definition should not adduce any new piece of information.

In order to solve the puzzle it is sufficient to mind the distinction between the object formal language the definiendum belongs to and its ‘metalinguage’ (hyperlanguage) the definition belongs to. I repeat: the piece of information adduced by a definition is a ‘meta-information’ (a hyper-information) concerning the assignation of an object information to a definiendum. A situation analogous to a direction teaching us how to consult a telephone book; it explains how to read the list of the users, yet it does not enrich the same list.

4.7.1. This dialinguistic distinction (also: this projective distinction) between the planes of the definiendum and of the definition explains immediately why the syntactic status of the definiendum does not influence the syntactic well formation of a formal definition. For the sake of concision I limit to definitions by signification a discourse which can be immediately referred to definitions by semantic identity or by synonymy). For instance

(4.xxiv) by definition “femur” means *thigh bone*

(4.xxv) by definition “to susurrate” means *to speak softly*

(4.xxvi) by definition “otherwise” means *in a different manner or by other means*

are syntactically well-formed hyperlinguistic sentences concerning respectively a substantive, a verb and an adverb (of the object language). As we already know (§2.5), the syntactical well formation of (4.xxiv), (4.xxv) and (4.xxvi), notwithstanding the different syntactical status of the three definienda, depends on the fact that quotation marks and asterisks are dialinguistic symbols substantivizing (in the dialanguage) whatever expression (of the object language) they enclose.

From this viewpoint the distinction between explicit and contextual definitions (for instance Pap 1964 in Olszewsky 1969, p.285) is of no theoretical moment, since we can consider all definitions as contextual and remark that there are contexts where some syntactical simplification is attainable. In particular the distinction emphasized by Pap's examples on -brother- is influenced by the double reading discussed in §19.7 (the brotherhood function, so to write, is not its generic 'value').

4.8. A traditional (yet naive) distinction opposes nominal and real definitions (that is, in Ockham's terminology, *definitiones quid nominis* and *quid rei*). This distinction is currently explained by statements like

*Real definition is distinguished from nominal definition as
being the definition of a thing rather than of a notation*

(Church under the voice "Definition" in Encyclopædia Britannica 1963) or like

*A real definition is understood to define the object for which
a term is used, a nominal definition to define the term ...*

(Olszewsky 1969, footnote p.280). I dissent because of a basic difficulty: what on earth do
the definition of a thing

or

to define an object

mean? Once any metaphoric interpretation under which, say, "to define" becomes a synonym of "to make" or of "to design" is banned,

to define a hat

is an improper expression. Logicians are not jaguars, but neither hatters; what they can define is the informational import of a word, not the shape of a garment.

Let me apply Carnap's well known distinction between material and formal modes of speech to a topic where it is actually enlightening, that is to definitions. Then

(4.xxvii) a pentagon is a polygon with five sides

is the material version and

(4.xxviii) "pentagon" means *polygon with five sides*

et cetera are the formal versions of the same definition. But what we can introduce in a formal system are words, not geometric figures: (4.xxvii) is nothing but a pseudo-objective way to express the piece of information adduced by (4.xxviii) (or by (4.xxix) et cetera). In my opinion 'real definitions' are nothing but definitions formulated in the material mode of speech.

4.8.1. Incidentally. So far as his original examples are concerned (1935, 7), Carnap's distinction is untenable. To read

(4.xxix) This book treats of Africa

as a pseudo objective sentence for the formally correct

(4.xxx) This book contains the word "Africa"

is a manifest far-fetched claim: *being treated in this book* is a connotation (*a quality*) of Africa exactly as *being visited by Mr. A*.

Furthermore, while the alethic values of (4.xxvii) and (4.xxviii) must be the same, the alethic values of (4.xxix) and (4.xxx) may be different.

The informational approach is also fit for avoiding another analogous mistake. In fact he claims that

(4.xxi) (the words) "Morning-star" and "Evening-star" are synonymous

is the formal mode of speech through which we can translate the pseudo object sentence

the Morning-star is the Evening-star

(material mode of speech). I dissent as I think that his position does not account for a basic informational discrepancy. In fact, for instance, as soon as we compare (4.xxi) with

(4.xxi) (the words) "femur" and "thigh bone" are synonymous

we can realize that while in (4.xxi) the two mentioned expressions refer to the same informational nucleus (to the same image, so to say briefly), in (4.xxi) they do not, since the piece of information we draw from the sight of the Morning-star is absolutely different from the piece of information we draw from the sight of the Evening-star. In other words. While, until the discovery of their identity, "Morning star" and "Evening star" named two different heavenly bodies, never "femur" and "thigh bone" named two different bones; in this sense they are connected by a very synonymy.

As soon as we realize that a third protagonist (the meaning) does exist between sign and referent, statements of identity are no longer puzzling. Anyhow I shall retake this crucial passage.

4.8.2. Ajdukiewicz (1958) writes: *there is no general concept of definition of which the concepts of real definition and nominal definition would be specifications the word "definition".... has in isolation no meaning at all.* Indeed, on the ground of the proposed considerations, Ajdukiewicz's claim seems to me too hazardous to be tenable (all the tenable hazardous claims I know are exactly mine). The word "definition" has in isolation a precise meaning since a definition is any intervention supplying the piece of information necessary to assign a meaning to a definiendum, quite independently on the means through which such a piece of information is supplied (let me evoke again the mother uttering "hat", "hat", "hat"). His too hazardous claim is born by the lacking informational dimension inducing him to over-estimate unessential peculiarities.

4.9. Definitions are actually classified through paradigms inspired by numerous and heterogeneous criteria. Yet many of these criteria can be neglected because they are superficial enough to result of no theoretical interest.

A current intriguing criterion (for instance Wikipedia, under the same voice) opposes descriptive definitions (which refer to *the general use*) to stipulative ones (which refer to *the speaker's immediate intentional meaning*). So, while (4.xxvii) instances a descriptive definition (for the notion of a pentagon belongs to the *general use*),

(4.xxxiii) a pentahusband is a man with five wives
 instances a stipulative definition (for, as far as I know, the notion of a pentahusband has just been introduced by my personal intervention, that is by (4.xxxiii)). Of course, in spite of its vagueness, the opposition between descriptive and stipulative definitions concerns different informational situations and as such it deserves a more systematic analysis. For the sake of concision, henceforth I only consider definitions by signification.

4.10. Let me scrutinize the matter from the viewpoint of the interpreter. The piece of information he draws from a given definition, that is from the acquirement of the signification through which a certain meaning is assigned to a certain definiendum, depends on what the same interpreter previously knows about such a signification (two different interpreters or the same interpreter in different moments may draw different pieces of information from the same definition). If we schematically and propaedeutically assume that these possible cognitive situations are only two (*known* vs. *unknown*), such situations are four, and precisely

- both definiendum and meaning are previously unknown (total definitions)
- the only meaning is previously known (christening definitions)
- the only definiendum is previously known (connotative definitions)
- both definiendum and meaning are previously known (heuristic definitions).

Indeed under the mentioned assumption (*known* vs. *unknown*) heuristic definitions are border-line cases. In fact if both *correlata x* and *y* are previously known, there is no informational gap to fill (the definition does not increase the statute). Yet as soon as we refine the opposition, so accounting for the thousand degrees of our actual knowledge about a topic, we can reasonably refer heuristic definitions to cognitive situations where a previously gross signification is better specified (§4.12).

4.10.1. The basic achievement is that, according to the above approach, the status of a definition varies with the statute of the interpreter; an unquestionable relativization, yet, because what can be a total definition for Plato, say, may be a christening definition for Socrates.

Getting over this relativization by making reference to the (indeed vague) figure of an average interpreter (whose statute corresponds just to the current knowledge about the topic involved by the definition under scrutiny) is restoring the (indeed vague) opposition between descriptive and stipulative definitions.

4.11. In order to emphasize the dependence of the classification on the statute of reference, in the first example, instead of considering different interpreters, I consider the same interpreter in different cognitive situations. So let

(4.xxiv) "duagon" means *(regular) polygon with two sides*
 be the example under scrutiny (incidentally: the parentheses of (4.xxiv) say that the specification of regularity is superfluous because, as we shall see, irregular polygons with two sides do not exist).

4.11.1. Total definition. Plato never met "duagon" and never minded the notion of a polygon with two sides, then (4.xxiv) defines a signification concerning a previously unknown term and a previously unknown meaning.

4.11.2. Christening definition. Plato was previously convinced that Euclide's notion of a polygon was too restrictive, since the case of $n=2$ is neither considered; but the same Plato never coined a name for this new geometrical figure, nor met such a name before facing (4.xxiv), which then assigns a previously unknown name to a previously known notion.

4.11.3. Connotative definition. The advanced essay on plane geometry Plato is studying speaks of duagons without any explanation.

- What is a duagon?

he asks Socrates. And the master:

- By definition a duagon is a regular polygon with two sides.

4.11.4. Heuristic definition. Plato replies:

- But what on earth is a regular polygon with two sides?

And the master:

- Have you a clear idea of a polygon with three or more sides?

And run along its perimeter, we come back to the point we started from.

- Do you agree that a polygon with $n > 2$ sides is regular iff it is equiangular and equilateral?

And Plato, after a quick reflection

- Yes, I do.

- Do you agree that the common value θ of its n internal angles is $(n-2)180^\circ/n$?

And Plato, after meticulous computations

- Yes, I do.

- Do you agree that, therefore, in a duagon the internal angles must be two, both of 0° ?

- Yes, I do.

Do you agree that, once fixed a point as the starting vertex and an arbitrary segment as the first side, to follow with a 0° internal angle is to come back towards the starting vertex?

- Yes, I do,

Do you agree that a segment in going and coming back binds a closed plane figure (although with a null surface)?

- Yes, I do.

- Do you agree that such a figure satisfies all the mentioned requisites of a regular polygon?

- Yes, I do.

Therefore you have a precise geometrical idea of what a regular polygon with two sides is. And indeed it complies too with our gross intuition according to which a regular polygon with n sides is the most regular polygonal arrangement of a regular polygon with $n+1$ sides once we erase one of its sides.

But are we sure that also the other quantities characterizing the notion of a regular polygon apply as well to a duagon?

As far as I know, they do. For instance the notions of inscribed and circumscribed circumferences survive respectively in the middle point of the segment and in the circumference admitting it as a diameter. For instance the sum of its external angles (720°) continues satisfying the general formula $n(360^\circ - \theta)$. And so on.

- I understand: a regular polygon with two sides is a couple of identical and superimposed segments.

4.12. Besides the Socratic maieutics, heuristic definitions play a momentous role in the history of the human culture because many fundamental philosophic problems can be conceived as the research for an heuristic definition. The best example is Tarski's research for a definition (**definition**, I emphasize) of "truth". Awaiting it (Chapter 5), I can remind St. Augustine's reflection about *time*, or, even better, Frege's studies on a satisfying definition (**definition**, I re-emphasize) of "number". Also before the publication of his *Die Grundlagen der Arithmetik*, "number" and *number* were respectively a word and a notion of consolidated use (thence the heuristic character of the pursued definition), yet any attempt failed to answer the question: what is a number? For instance, the purpose of the mathematician who claimed that "number" was nothing but a synonym of "difference" was to find out and display the connotation that, although hidden in the semantic tissue of ordinary language, is essential in *number*. And in order to understand that his claim is untenable it is sufficient to remember the *bon-vivant* who would be struck with horror at the idea that his "*Vive la difference!*" should be interpreted as an ode to the number.

4.13. Let me insist through an example concerning a translation dictionary (in its two sections English-Italian and Italian-English, say). The manifest ellipticity of a formulation like

(4.xxxv) **meddlesome** invadente

is inconsequential because, since he who consults a dictionary is previously aware of the context (is previously aware that he is dealing with definitions), simplified graphic conventions are legitimate. And exactly the ellipticity of (4.xxxv) allows its reading as a definition by signification, or indifferently by semantic identity, or by synonymy; these readings follow directly from the interpretation of the mentioned graphic conventions.

In all cases the same title of the volume under consultation ("English Italian Dictionary", say), through its first word tells us that the first word of (4.xxxv) belongs to the lexicon of English, therefore that $L1$ is English, and through its second word tells us that $L2$ is Italian. Under a reading by signification (I evoke (4.xiii)) the boldface acts as a couple of inverted commas, the following space acts as a "σ", the Roman type acts as a pair of asterisks. On the contrary, under a reading by semantic identity, both boldface and Roman type act as asterisks, the space between them acts as a predication of (semantic) identity and so on.

Now I analyze (4.xxxv) according to the cognitive endowment of four different readers.

4.13.1. First reader. My deeply uncultured Italian friend Jim, glancing inattentively through the English-Italian section, meets casually (4.xxxv); thus he learns that this previously unknown “meddlesome” means in English the previously unknown ${}^*_{invadente}{}_I$, and fills his latter informational lack by asking me what “invadente” means in Italian. For Jim (4.xxxv) is then a total definition.

4.13.2. Second reader. My semi-cultured English friend Ted, who knows the meaning of “meddlesome”, consults that voice to learn the Italian adjective such a meaning is adduced by. For Ted (4.xxxv) is then a christening definition telling him that “invadente” is the Italian adjective he is searching for.

4.13.3. Third reader. My semi-cultured Italian friend Bob meets “meddlesome” while he is reading an English novel and, since he does not know its meaning (since he does not know ${}^*_{meddlesome}{}_E$), he brings himself to consult the voice “meddlesome” in the dictionary. For Bob (4.xxxv) is then a connotative definition telling him that the well known ${}^*_{invadente}{}_I$, is the meaning to correlate with the well known “meddlesome” he was starting from.

4.13.4. Fourth reader. My superficially polyglot friend Tom knows vaguely that ${}^*_{meddlesome}{}_E$ is a rather negative connotation, then he consults the dictionary just to improve this fuzzy signification. For Tom (4.xxxv) is then a heuristic definition.

4.13.5. Concisely. The English-Italian section, normally, is a book of christening definitions when in English hands, and a book of connotative definitions when in Italian hands. Vice versa, obviously, as for the Italian-English section.

4.14. Another interesting and non-fictitious example concerns ${}^*_{planet}$. The four contexts can be clearly singled out, as ancient astronomers observed that seven heavenly bodies were in apparent wandering motion with respect to the fixed stars, and agreed to call them “planets” (“πλανῆς” $GR \leftrightarrow E$ “I wander”), but just in our days a congress of astronomers is discussing the precise connotations a heavenly body must possess in order to be classified as a planet. While this congress is pursuing a heuristic definition, the absolutely incompetent Plato who, reading an astronomical report, asks Socrates “What is a planet?” is pursuing a connotative definition et cetera.

4.15. The tripartition of §4.3 and the tetrapartition of §4.10 are completely independent. The latter can be applied to definitions by semantic identity or by synonymy, but even to ostensive definitions. For instance the mother uttering “hat” is performing

- a total definition if the child ignores both the notion of a covering for the head and the uttered word
- a christening definition if her son already knows the notion but ignores the word,
- a connotative definition if the child asked her what is a hat (he knows the word but ignores its meaning)
- a heuristic definition if the child is learning that there are covering for the head which are not hats, but caps, helmets et cetera.